1 INTRODUCTION

In a previous paper\(^1\) we proposed an experimental method for the determination of the binaural acoustic pressure signals generated during bat echolocation. A part of that modelling method involved the determination of the backscattering impulse response for a target of a given shape, position, orientation and material. This paper focuses on the problem of determining such backscattering impulse responses for rigid disc-shaped objects. We present experimental results from an extensive dataset of measurements that we have collected and we compare them with analytically predicted impulse responses computed using a previously proposed time-domain analytical model\(^2\).

2 EXPERIMENTAL METHOD

In the following we build on our previously presented formulation\(^1\) in which the backscattering process is modelled as a linear and time-invariant system in discrete time. The objective is then to determine the impulse response \(h_{\text{backscatter}}\) that relates the acoustic pressure signal \(p(n)\) generated at a point, say \(r\), when a source insonifies an echo-generating object, to the acoustic pressure signal \(p_{\text{ff}}(n)\) at a point, say \(r_{\text{ff}}\), generated when the same source radiates sound in the free field. This relation is described by the convolution of equation (1) in which the dependence of \(h_{\text{backscatter}}\) on the point \(r\), the type of the backscattering object (shape, material, etc.) and its orientation relative to the source are specifically noted.

\[
p(n) = h_{\text{backscatter}}(r, \text{object, orientation}, n) * p_{\text{ff}}(n)
\]

For the purposes of the presentation in this paper, the source, the echo-generating object and the points \(r\) and \(r_{\text{ff}}\) are taken to be in line and we use the scalar distances \(r\) and \(r_{\text{ff}}\) from the echo-generating object to describe the geometry.

2.1 Measurement setup

Our measurement setup is described in part (a) of Figure 1. A source is driven by the electrical input signal \(v_{\text{in}}(n)\) generating an outgoing acoustic pressure signal \(p_{\text{out}}(n)\) at point \(r\). The acoustic wave reaches the target and an echo is formed that generates the pressure signal \(p_{\text{echo}}(n)\) back at point \(r\). A sensor at point \(r\) will pick up the superposition of these two signals. The same setup but without an echo-generating object in place can record the free field signal \(p_{\text{ff}}(n)\) generated at point \(r_{\text{ff}}\) when the source is driven by the same input signal \(v_{\text{in}}(n)\).

For the measurements presented here, the input signal \(v_{\text{in}}(n)\) was a pseudorandom maximum length sequence at 160kHz sampling rate generated by a PC equipped with a MLSSA\(^3\) board and amplified by an audio power amplifier (Yamaha H5000). The source was an off-the-self piezoelectric tweeter with quoted upper frequency cut-off at 40kHz and the microphone a \(\frac{1}{4}\)"
B&K4135 connected to a B&K2610 measuring amplifier. The captured signal was fed back to the MLSSA board and the overall system’s impulse response between the input $v_i(n)$ and the output from the measuring amplifier was computed by means of cross-correlation. Based on the above specification, the usable frequency bandwidth of our setup was 3-35kHz (for more details see1). The apparatus described in part (a) of Figure 1 was positioned inside ISVR’s small anechoic chamber (part (b) of Figure 1) with the controlling and/or noisy equipment (PC and power and measuring amplifiers) positioned outside the chamber.

Figure 1: (a) Schematic of the experimental setup, (b) Experimental rig in ISVR's small anechoic chamber

We note here that for practical reasons, the value of the distance parameter $r$, was chosen during the experiments to correspond to the distance of the bracket supporting the microphone from the frame supporting the echo-generating object, whereas the actual distance of the centre of the microphone diaphragm from the plane containing the axis of rotation of the echo-generating object was a further 39mm from the value noted here as $r$. For clarity of presentation we keep here the originally noted values of the distance parameter $r$ (which amount to multiples of 5cm) but use the exact distance values in the computation of the analytically predicted results presented below. We also note that the measurements presented here were obtained with an earlier version of the measurement rig in which the echo-generating object was rotated manually rather than with the motorised mechanism depicted in Figure 1. We estimate the accuracy of the manual rotation to be approximately +/-2°. We expect this error to partly account for any discrepancies between measured and predicted results presented below.

2.2 Analytical model

The analytical presented results presented here were all obtained by use of a Matlab realisation of the time-domain edge diffraction model provided by Prof. U. P. Svensson. Even though the modified Biot-Tolstoy-Medwin theoretical model described in reference is also applicable to curvilinear edges, the Matlab code we obtained from Prof. U. P. Svensson is restricted to diffraction off straight edges. Hence for the results presented here, the circular edge of the discs used was approximated by polygons. By repeating the computation with increasing order polygon approximation we found that beyond 256-corner approximations resulted in no significant difference in the computed impulse responses, so we kept this value for our results. We are currently working for the adaptation of the Matlab code to curvilinear edges. The computations were run for 160kHz sampling rate and only first order diffraction was computed. Determining the actual position of the sound source with accuracy was not straightforward. By comparing our measured and analytically predicted results for a single specular reflection (see §3.1) we estimated this distance to 60.25cm from the plane containing the axis of rotation of the echo-generating object and we used this value as $p$ or $p_{ff}$.
for the position of the source in all our analytical predictions. A correction of 39mm was applied to the quoted distance of the receiver the plane containing the axis of rotation of the echo-generating object as described above.

3 RESULTS

3.1 Specular reflection

In part (a) of Figure 2 we plot the impulse responses measured with a 1m² flat aluminium sheet placed at the position of the echo-generating object and with the microphone positioned at a distance \( r \) of 35cm, 30cm, 25cm, 20cm and 15cm from the echo-generating object. The delay in the part of the impulse response corresponding to \( p_{\text{out}} \) and the advance of the part corresponding \( p_{\text{echo}} \) can be clearly seen as well as the decay due to spherical spreading. For a perfect specular reflection, this decay is expected to follow a \( 1/r \) trend, something with which part (a) of Figure 2 is in rough agreement. We note that for an exact evaluation of this effect we would need to upsample our data in order to estimate the exact peak values. However, for such an exact analysis, the effect of atmospheric attenuation might also have to be taken into account and possibly more accurate results could be obtained with a better manufactured reflector. Hence, we leave the analysis pertinent to the exact strength of the returned echo outside the scope of this paper and we focus our attention here in the shape of the echo signature.

![Figure 2](image)

**Figure 2**: (a) Raw impulse response measurements between input \( v_{\text{in}} \) and output captured in the microphone for \( r=35\text{cm} \) (blue line), \( r=30\text{cm} \) (red line), \( r=25\text{cm} \) (green line), \( r=20\text{cm} \) (black line) and \( r=15\text{cm} \) (magenta line). (b) Same as part (a) but equalized with respect to the free field measurement at \( r=35\text{cm} \). The sample number index is aligned to 0 at the time when the direct wave field passes the point \( r=35\text{cm} \).

Evidently, the results of part (a) of Figure 2 deviate from the ideal pair of delta pulses that would correspond to an ideal source, receiver and data acquisition system as they contain a long and energetic decay tail due to the combination of the individual responses of the measurement equipment. In part (b) of Figure 2 we plot the results obtained when those raw responses are equalised with respect to the free field measurement obtained with the microphone positioned at the position \( r=35\text{cm} \) and with no echo-generating object in place. The equalisation is effected by means of frequency division of the two responses and subsequent linear-phase bandpass filtering of the result by a 3-30kHz bandpass filter (the linear-phase filtering is computed using Matlab’s `filtfilt` forward-backward time filtering command). Barring the time-symmetric smearing of the response due to the linear-phase bandpass filtering, the equalised measured responses demonstrate the expected delta pulse characteristics of the ideal measurement system.
Referring back to the LTI model described by equation (1), convolution of a given signal $p_{rec}$ recorded at point $r_f$ in the free field with the impulse response $h_{\text{backscatter}}(r, \text{object}, \text{orientation})$ corresponding to a point $r$, will yield the acoustic pressure signal created at point $r$ when a specific object placed at the specific orientation is insonified by the same source. In the case where the echo-generating object is not just an acoustic mirror, but a more complex object (as are the cases examined in the next sections), the part of the impulse response corresponding to the echo will not be a delta pulse but a more complex signature.

In part (a) of Figure 3 we plot the result obtained from the direct application of the code described in §2.2 to the five geometries that resulted in the measurements of Figure 2. The only effect modelled by the code is that of a specular reflection resulting in an ideal delta pulse of infinite bandwidth. Sampling this pulse at any sampling rate is bound to misjudge its peak value, as is evident by the results of the figure. In part (b) of Figure 3 we plot those same results being processed by the same linear-phase bandpass filter that was applied to the equalisation process of part (b) of Figure 2. A single scaling is applied to all results of part (b) of Figure 3 equating the height of the direct wave pulse (blue line at $n=0$) to the amplitude of the same sample in the measured result of part (b) of Figure 2. The thus conditioned analytical prediction can be seen to be in very good agreement with the measured results of part (b) of Figure 2.

![Figure 3](image)

**Figure 3:** (a) Prediction of the analytical model for the same geometries as Figure 2. (b) Same as part (a) after linear-phase bandpass filtering. The sample number index is aligned to 0 at the time when the direct wave field passes the point $r_f=35cm$ and a common scaling is applied equating the height of the direct wave pulse to the measured result of Figure 2.

All the results presented below are obtained in the same manner as those of parts (b) of Figure 2 and Figure 3. That is, the measured results are equalised by dividing in frequency with the free field response obtained with the microphone at $r_f=35cm$ and then linear-phase bandpass filtered, whereas the analytically predicted results are just linear-phase bandpass filtered.

### 3.2 Disc orientation and size

In Figure 4 we plot the measured and analytically predicted results obtained with a perspex disc of 10cm diameter and 1.5cm thickness as the echo-generating object in 7 different rotations, from 0° (flat face facing the source and receiver) to 90° (circular perimeter facing the source and receiver). As in all subsequent plots, the axes are zoomed in the part of the impulse response corresponding to the echo. Barring the exaggerated prediction of the echo strength for the 0° rotation, very good agreement can be seen between measurement and analytical prediction. More specifically, the timing, individual shape and strength of the leading and trailing edges of the disc as they move closer and further away from the source is in copied faithfully from measurement to prediction. The
results obtained from a perspex disc of 6cm diameter and 1.5cm thickness are plotted in Figure 5. In this instance, the shorter distance between leading and trailing edge is clearly visible equally in measurement and prediction. Similar backscattering signatures (but in a slightly different geometric arrangement) have been previously measured\textsuperscript{6} for bat echolocation studies but only providing qualitative rather than analytically exact descriptions of the impulse response shape.

Figure 4: Backscattering impulse response of a perspex disc of 10cm diameter and 1.5cm thickness for various angles of incidence, (a) measurement (b) analytical prediction.

Figure 5: Backscattering impulse response of a perspex disc of 6cm diameter and 1.5cm thickness for various angles of incidence, (a) measurement (b) analytical prediction.

In Figure 6 we plot the results obtained for a plastic disc of 1.5cm diameter and 0.2cm thickness. Most features of measurement and computation are still in agreement, but measurement and processing noise in the experimental results are of comparable magnitude, thus placing this size of echo-generating object at the margin of reliability for our measurement setup. We are currently upgrading our measurement rig for higher positioning accuracy and an upper frequency bandwidth limit of 100kHz, thus expecting to be able to reliably resolve backscattering impulse responses of smaller objects in the near future.
3.3 Disc thickness

In Figure 7 we plot and compare the results obtained for two plastic discs of equal diameter (6.4cm) but different thickness (2cm and 1cm respectively). Both measurement and analytical prediction consistently show the leading echo off the thicker disc at 0° but are in questionable agreement in showing the differences due to the differently spaced double edge as it rotates towards the source. It can be seen, however, that most points of discrepancy between measurement and analytical prediction have to do with the timing of specific features rather than their individual shape. We expect that this type of result can be significantly improved when the measurements are repeated with the motorised rotation mechanism installed on our rig. This would allow our measured impulse responses to successfully resolve this quite subtle feature.

Figure 7: Backscattering impulse response of a plastic disc of 6.4cm diameter and 2cm thickness (blue line) and of 6.4cm diameter and 1cm thickness (red line) for various angles of incidence, (a) measurement (b) analytical prediction.
4 DISCUSSION – CONCLUSION

The significance of the results we presented lies in the fact that they cross-validate two independent modelling methods. Hence, even though restricted to rigid objects of simple shape, the modified Biot-Tolstoy-Medwin formulation presented in\(^2\) proves a reliable method for the prediction of the detailed form of the individual backscattering impulse response signature of such objects. The main advantages of such a tool for the purposes of our echolocation modelling research are that (i) it is computationally very cheap and easily scalable to wider frequency bandwidths (ii) being formulated entirely in the time-domain, is allows direct interpretation as well as inclusion or exclusion of the contributions of specific glints of the echo-generating object (iii) it is devoid of the noise and processing artifacts of measured results (particularly the non-causal ringing due to the linear-phase bandpass filtering). On the other hand, the agreement of our equalised measurement results with the analytical prediction validates our measurement and equalisation setup up to its current degree of positioning accuracy and frequency-related resolution capability and hence renders it a reliable method for the determination of the backscattering impulse response of more complex and non-rigid objects. We are currently working on a computation scheme for the curvilinear edge formulation, on the upgrade of our positioning mechanism to a fully motorised version and on the replacement of our source and data acquisition equipment in order to increase the usable bandwidth of our setup to an upper limit of 100kHz.

5 ACKNOWLEDGEMENTS

The analytical prediction results presented in this paper were computed using Matlab code provided by Prof. U. P. Svensson and this is gratefully acknowledged. The authors are also very grateful to RCUK for support through the BIAS Basic Technology Programme and acknowledge the contribution of team members in the many discussions along the way.

6 REFERENCES